A Model of Budget Constraint and Enterprise Restructuring

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Abstract

This paper presents a managerial utility maximisation framework to investigate the impact of budget constraints on enterprise restructuring in transitional economies. Assuming that managerial effort is allocated between restructuring and subsidy seeking, and using non-linear programming technique, we have demonstrated that hardening budget constraint by reducing state subsidies to state firms have positive effects on managerial incentives towards enterprise restructuring. Our results apply both to the perfectly competitive case and the monopolistic case. We have, however, uncovered a perverse relationship between labour demand and wage rate.

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1. Introduction

It has been a well-known fact that state subsidies to loss-making enterprises under central planning have resulted in the so-called soft budget constraint syndrome. While a great deal of attention has been paid to the consequences of the soft budget constraint in socialist economies, the withdrawal of state subsidies has received little attention. An inspection of a 1998 symposium organised by the *Journal of Comparative Economics* reveals that noone has dealt with the impact of hardening the budget constraint, given the prominent role it has played under central planning. Since transition commenced, state subsidies have been reduced drastically.\(^1\) This seems to have hardened budget constraints. The transition literature, however, tend to be more concerned with privatisation than with budget constraints.\(^2\) Qian and Roland (1998) analyse the impact of federalism on enterprise budget constraints. They have not examined its impact on enterprise restructuring, however. Aghion *et al.* (1993) earlier studied enterprise restructuring. They focused instead on downsizing, de-bundling and privatisation.

This paper, by contrast, considers the impact of budget constraints on enterprise restructuring. It models explicitly subsidy reduction and managerial effort allocation between subsidy seeking and restructuring. We are able to obtain a general result on the effect of hardening budget constraints on enterprise restructuring in transition economies. Section two formulates the managerial utility maximisation problem. Section three explores some properties on factor demand and managerial effort allocation mechanisms. Section four presents some comparative static results. We find that hardening budget constraints via limiting the amount of subsidy is definitely conducive to enterprise restructuring. We give some more specific examples to illustrate this proposition in Section five. Section six provides a summary.

2. Subsidy Seeking *versus* Restructuring

Evidence since transition indicates that as the state withdraws from the enterprise level, managers are taking control of the day-to-day operation of the enterprise.\(^3\) To focus on subsidy reduction and restructuring, we shall neglect this complication and, to simplify analysis, we assume that managerial effort is allocated between restructuring and subsidy seeking. It should be noted that subsidy seeking, as the legacy of planning and government control, is one particular form of influence activity, which occurs in numerous contexts.\(^4\) It certainly occurred in partially reformed systems. In his pioneering study of labour managed firms, Ward wrote: ‘While the Yugoslav economic system thus involves a considerable amount of

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\(^2\) Aghion *et al.* (1993); Boycko *et al.* (1996).

\(^3\) Estrin and Wright (1999).

\(^4\) Milgrom (1988); Coase (1959); Tullock (1967); Krueger (1974).
autonomy for the firm, it should not be thought that independence of the sort possessed within the legal framework of capitalism has not been acquired by the Yugoslav firm. The state reserves the right to intervene directly to alter any decisions of which it disapproves. The firm itself in its independent plan set its own goals for the year and even then was not penalised for failure to fulfil these targets.\textsuperscript{5}

Formally, restructuring output is determined by a typical neo-classical function:

\[ Q = h(N, e_r) \]  

where \( Q \) denotes restructuring output, \( N \) is the number of workers and \( e_r \) is managerial restructuring effort. The restructuring function is of the neo-classical type with positive but diminishing marginal products.

A distinguishing feature of this model is to assume that total obtainable subsidy is a fixed parameter for the enterprise, that is,

\[ S = F(e_r) \leq A \]  

where \( S \) is the subsidy obtained; \( A \) is obtainable subsidy and \( e_r \) is managerial subsidy-seeking effort. Total profit made is:

\[ \pi = \phi(Q).Q + S - wN \]  

where \( \phi(Q) = p \) is the inverse demand, \( Q \) is restructuring output, \( S \) is subsidy obtained, \( w \) is wage rate and \( N \) is employment.

Managerial effort allocation between restructuring and subsidy seeking is as follows:

\[ e = e_r + e_i \]  

If the manager is concerned with his effort and profitability, the utility function may be written as:

\textsuperscript{5} Ward (1958, p. 567, 570).
\[
U = U(\pi, e)
\]

\(U_\pi > 0, U_{\pi e} < 0, U_2 < 0, U_{22} > 0\). This means that marginal utility is positive but diminishing with profit and negative but increasing with total effort. The manager’s problem may then be formulated as follows:

**Max** \( U = U(\pi, e) \)

s.t.

\[
\pi = \phi(Q)Q + S - wN
\]

\(e = e_r + e_i\)

\(Q = h(N, e_r)\)

\(A \geq S = F(e_i)\)

### 3. Interpreting First Order Conditions

Because there is an inequality constraint in (5), a non-linear programming technique is applicable. Two cases are to be considered: if the manager chooses to stay within the subsidy limit, the subsidy constraint would stand as a strict inequality and the multiplier associated with this inequality will be zero. On the other hand, if the manager decides to obtain as much subsidy as possible, the constraint becomes an equality constraint. The multiplier associated with this case will be positive, as the Kuhn-Tucker Theorem requires.6

The Lagrangean is:

\[
V = U(\pi, e) + \lambda(pQ + S - wN - \pi) + \mu(h(N, e_r) - Q)
\]

\( + \nu(F(e_i) - S) + \sigma(e - e_r - e_i) + \tau(A - S) + \eta(\phi(Q) - p)\)

The first-order conditions are:

\[
\begin{align*}
U_\pi &= \nu + \tau \\
-U_e &= \sigma \\
\lambda Q &= \eta \\
\lambda P - \mu + \eta \phi_Q &= 0 \\
w\lambda &= \mu h_N \\
v_f &= \mu h_r
\end{align*}
\]

The complementary slackness condition dictates that if $S < A$ the multiplier $\tau$ is zero, if $S = A$ the multiplier $\tau$ is positive.

**3.1. $S < A$**

In this case, increasing the subsidy limit does not affect managerial subsidy-seeking effort and $\tau$ is zero. (7) Becomes:

$$
\begin{cases}
\lambda = v = U_x \\
\sigma = -U_e = \mu h_e = v F_e \\
\eta = U_x Q \\
U_x p - \mu + U_x Q \phi_Q = 0 \\
\mu = \frac{w U_x}{h_N}
\end{cases}
$$

(8)

Or

$$
\begin{align*}
(a). \quad \frac{h_e}{F_e} &= \frac{h_N}{w} \\
(b). \quad \frac{U_e}{U_x} &= w \frac{h_e}{h_N} \\
(c). \quad w &= h_N (\phi(Q) + Q \phi_Q)
\end{align*}
$$

In (9), $F_e$ refers to the marginal product of subsidy-seeking effort; $h_e$ denotes the marginal product of restructuring effort; $h_N$ denotes the marginal product of workers; $U_e, U_x$ denote the marginal utilities of managerial effort and profit; $\phi(Q)$ is the inverse demand function and $\phi_Q$ denotes infinitesimal change of price due to an infinitesimal change of output. It measures the movement along the demand curve. It therefore specifies the degree of monopoly in the product market.

The left-hand side of (9a) is the ratio between the marginal products of restructuring and subsidy seeking. It measures the technical rate of substitution between the effort of restructuring and the effort of subsidy seeking. The right hand side of (9a) is the ratio of the marginal product of labour to the on-going wage rate in the labour market. In equilibrium they must be equal. Thus (9a) gives the optimal rule for managerial effort allocation and employment.

The left-hand side of (9b) states the marginal rate of substitution between profit and the managerial effort. The right hand side of (9b) is the product of two terms: one is
the wage rate and the other is the technical rate of substitution between the managerial effort of restructuring and labour. (9b) says that the marginal rate of substitution between profit and managerial effort must be proportional to the technical rate of substitution between managerial effort of restructuring and labour the proportionality factor being the wage rate.

The left-hand side of (9c) is the wage rate. The right hand side is a product of two terms. One is the marginal product of workers and the other is just the marginal revenue. The first term in the parenthesis on the right hand side is the product price and the second term is the product of the output and the influence of output on the market price. If the firm is a price taker, the second term in the parenthesis would vanish, leaving us with the usual marginal value product condition.

(9a)-(9c) together imply:

\[ -\frac{U_e}{U_\pi} = F_e \]  

(9)' has a simple interpretation: if the subsidy limit is not reached, the marginal rate of substitution between total managerial effort and profit is equal to the marginal productivity of subsidy-seeking effort.

3.2. \( S = A \)

\( \tau \) in this case is positive since the manager chooses to obtain maximum subsidy. Subsidy-seeking effort is now determined by \( S = F(e_\tau) \) and (7) now becomes:

\( h_e > \frac{h_N}{F_e} \)

\( -\frac{U_e}{U_\pi} = w \frac{h_e}{h_N} > F_e \)  

\( w = h_N (\phi(Q) + Q\phi_Q) \)

Or

\[ -\frac{U_e}{U_\pi} > F_e \]  

(10)'

The interpretation of (10)' is if the subsidy limit is reached, the marginal rate of substitution between total managerial effort and profit is greater than the marginal productivity of subsidy-seeking effort.

(9)' and (10)' imply the following theorem:
Theorem

If the subsidy limit is reached \((S = A)\), the marginal rate of substitution between total managerial effort and profit must be equal to the marginal productivity of subsidy-seeking effort; if the subsidy limit is not reached \((S < A)\), the marginal rate of substitution between total managerial effort and profit must be greater than the marginal productivity of subsidy-seeking effort. This may be expressed as:

\[
\frac{U_e}{U_\pi} \geq F_i
\]  

(11)

Throughout this paper we assume that the second order conditions are met.

4. Comparative Statics

To obtain specific comparative statics results, we use the following functional forms:

\[
\begin{aligned}
U &= \pi + e^{-1} \\
h &= N^a e^{1-a} \\
F_i &= be_i \\
P &= c - dQ
\end{aligned}
\]  

(12)

Again two cases are distinguished. If the subsidy limit is not reached, (9) becomes:

\[
\begin{aligned}
\frac{1-a}{b} &= ae_r \\
e^2 &= \frac{1}{b} \\
w &= acN^{a-1}e_i^{1-a} - 2adN^{2a-1}e_r^{2(1-a)}
\end{aligned}
\]  

(13)

Or

\[
\begin{aligned}
N &= \frac{c}{2d} \left( \frac{1-a}{ab} \right)^{a-1} w^{a-1} - \frac{1}{2ad} \left( \frac{1-a}{ab} \right)^{2(a-1)} w^{2a-1} \\
e_i &= \frac{c}{2d} \left( \frac{1-a}{ab} \right)^a w^a - \frac{1}{2ad} \left( \frac{1-a}{ab} \right)^{2a-1} w^{2a} \\
e_r &= \frac{\frac{1}{b}}{2d} \left( \frac{1-a}{ab} \right)^a w^a + \frac{1}{2ad} \left( \frac{1-a}{ab} \right)^{2a-1} w^{2a}
\end{aligned}
\]  

(13)'

Proposition 1

It is clear from (13)' that labour demand decreases with wage rate if and only if the technical rate of substitution between labour and managerial effort is more than half.
Proof

Partial differentiation of \( N \) with respect to \( w \) in (13)' gives \( \frac{\partial N}{\partial w} < 0 \iff a \geq \frac{1}{2} \). QED.

Remark

Proposition 1 is consistent with a profit-maximising firm. However, if the substitutability between labour and managerial effort is less than one-half, employment may rise as the market wage rate increases. One possible explanation for this perversity is that because total subsidy is limited, rising wage rate entails higher cost, to compensate for higher cost associated with rising wage rate, restructuring activity must rise. This occurs when substitutability between labour and managerial restructuring effort is less than one-half. This case merits further investigation.

Proposition 2

If an enterprise is a price taker in the product market, labour demand falls as product price falls, i.e., \( \frac{\partial N}{\partial c} > 0 \).

The proof follows readily from partial differentiation of \( N \) with respect to \( c \) in (13)'.

Proposition 3

If the technical rate of substitution between labour and managerial effort is less than one half, lower efficiency in subsidy seeking enhances restructuring effort.

Proof

Differentiating \( e_r \) with respect to \( b \) in (13)' gives:

\[
\frac{\partial e_r}{\partial b} = -\frac{ac}{2d} \left( \frac{1-a}{a} \right)^a w^a b^{-a-1} - \frac{1 - 2a}{2ad} \left( \frac{1-a}{a} \right)^{2a-1} w^{2a} b^{-2a} < 0 \iff a < \frac{1}{2}
\]

Remark

Under the above condition, an improvement in subsidy-seeking efficiency would lower managerial restructuring effort. Given the subsidy limit, however, low efficiency in subsidy seeking encourages managerial restructuring effort.

Proposition 4

There exists a critical wage rate below which it encourages restructuring effort.

Proof

\[
\frac{\partial e_r}{\partial w} > 0 \iff w < \left( \frac{1}{2} \right)^{\frac{1}{a}} \frac{ab}{1-a}
\]
**Remarks One**
Under constant returns to scale in subsidy-seeking activity, if technical rate of substitution between labour and managerial effort is half, restructuring effort and the wage rate will change in the same direction.

**Remarks Two**
Proposition 5 means that governments have some leeway to induce the manager to increase restructuring effort. If the manager has to make more journeys to the Treasury or to make more phone calls, this is all the better for restructuring.

**Proposition 5**
If wage rate is higher than a certain level, subsidy-seeking effort rises.

**Proof**
This follows from direct differentiation of (13)’:

$$\frac{\partial e_i}{\partial w} \geq 0 \iff w \geq \left(\frac{2ab}{ac}\right)^\frac{1-a}{a}$$

In particular, if $b = 1, \ a = \frac{1}{2}$, then $\frac{\partial e_i}{\partial w} > 0 \iff w > \frac{c^2}{14}$. The reason is that as wage rate rises, profit from restructuring decreases, it is more worthwhile to increase profit from subsidy.

We shall not attempt to provide empirical tests of some of these propositions. Instead, we shall present some examples and give some casual evidence in Sections five and six.

**5. Some Examples**
To gain more insights into the relationship between employment and restructuring effort, substitute (12) into (10):

$$\begin{align*}
\left\{ \begin{array}{l}
\frac{N}{e_r}(e_r + \frac{A}{b})^2 = \frac{a}{(1-a)w} \\
\frac{cN^{a-1}e_r^{1-a} - 2dN^{2a-1}e_r^{2(1-a)}}{a} = \frac{w}{a}
\end{array} \right.
\end{align*}$$

Although (14) is hard to solve analytically, by fixing some parameters, we may obtain some striking results.

We present four examples for $a = \frac{1}{2}$.
Example 1. If \( A = d = 0 \), (14) becomes:

\[
N = \frac{4}{c^2} \times \frac{1}{e_r^3}
\]  

(14)'

Employment is inversely proportional to the cube of the restructuring effort. Labour demand and effort supply functions are:

\[
\begin{cases}
    e_r = \frac{2}{c} \sqrt{w} \\
    N = \frac{c}{2} \left( \frac{3}{w} \right)^{\frac{3}{2}}
\end{cases}
\]

It is clear that labour demand increases with product price and decreases with wage rate. This is consistent with a profit maximising firm. Managerial restructuring effort is proportional to the square root of the wage rate for a given product price. On the other hand, given the wage rate, restructuring effort is inversely proportional to product price. Falling product price therefore unambiguously increases restructuring effort.

Example 2. If \( A \neq 0, b = 1, d = 0 \), (14) becomes:

\[
N = \frac{4}{c^2} \times \frac{e_r}{(e_r + A)^4}
\]  

(14)''

Labour demand and restructuring effort functions are:

\[
\begin{cases}
    e_r = \frac{2}{c} \sqrt{w} - A \\
    N = \frac{c}{2} \left( \frac{3}{w} \right)^{\frac{3}{2}} - \frac{c^2}{4} A w^{-2}
\end{cases}
\]

It is easy to show that, \( \frac{\partial N}{\partial w} < 0 \) \( \forall \ w < \frac{4}{9} A^2 c^2 \). Note again that labour demand does not fall monotonically as wage rises. Lower product price again encourages managerial restructuring effort.

Example 3. If \( A = \theta c \), \( b = 1 \), \( d = 0 \), (14) becomes:

\[
N = \frac{4}{c^2} \times \frac{e_r}{(e_r + \theta c)^4}
\]  

(14)'''

Labour demand and restructuring effort are:
Restructuring effort is an increasing function of the wage rate. If the subsidy limit becomes smaller than product price, restructuring effort increases. Employment falls if the following condition holds:

\[
\frac{\partial N}{\partial w} < 0 \quad \forall \quad w < \frac{9}{4\theta^2c^2}
\]

**Example 4.** If \( d \neq 0, A = 0 \), (14) becomes:

\[
Ne^2c^2\left(\frac{1}{4}c^2e_r - 2d - d^2N\right) = 1
\]

If manager faces a downward-sloping demand curve, restructuring effort satisfies a quadratic:

\[
we^2e_r^2 - 8wde_r - 4(d^2 + w^2) = 0
\]

It is easy to show that \( \frac{\partial e_r}{\partial d} > 0 \). As the slope of the demand curve becomes steeper, managerial restructuring effort increases. If the demand curve is horizontal, this case degenerates into the first example.

To summarise these examples, if an enterprise is a price taker, then reducing subsidy has unambiguous effects on managerial restructuring effort and employment; the relationship between product price and employment is negative whereas the relationship between product price and restructuring effort is positive. On the other hand, if an enterprise possesses market power and if no subsidy is granted, labour and managerial restructuring effort change in opposite directions.

6. Conclusions

A general and encouraging finding of this paper is that reducing state subsidy unambiguously encourages restructuring in a utility maximisation framework. If no subsidy were offered regardless of lobbying intensity, all the managerial effort would be devoted to restructuring. This finding vindicates the widely held view that hardening budget constraints is conducive to restructuring.
Our model could be extended in several directions. First, introducing strategic aspects may endogenise subsidy limit.\(^7\) Secondly, various governance structures may be introduced into the model. Managerial utility maximisation in our model amounts to absolute managerial control. This applies to some transition firms. But other firms may require different treatment. Thirdly, we have deliberately left privatisation out of the model. Privatisation changes the maximisation problem drastically as it alters both objective function and constraints. Aghion et al. (1993) and Boyko et al. (1996) suggest that privatisation will have positive effects on enterprise restructuring as it makes state interference more costly. Of course our assumption of the subsidy limit may itself be a consequence of privatisation. Song and Gao (1999) indicate that all effects of privatisation need not be positive.

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\(^7\) Schaffer (1988).


